

Exercise 108

Express the limit as a derivative and evaluate.

$$\lim_{\theta \rightarrow \pi/3} \frac{\cos \theta - 0.5}{\theta - \pi/3}$$

Solution

Recall the definition of a derivative.

$$f'(a) = \lim_{\theta \rightarrow a} \frac{f(\theta) - f(a)}{\theta - a}$$

The function in question is

$$f(\theta) = \cos \theta.$$

Take the derivative.

$$f'(\theta) = -\sin \theta$$

Plug in $\theta = \pi/3$.

$$f'(\pi/3) = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

Therefore,

$$\lim_{\theta \rightarrow \pi/3} \frac{\cos \theta - 0.5}{\theta - \pi/3} = -\frac{\sqrt{3}}{2}.$$